If R, r, r_1, r_2, and r_3 be, respectively, the radii of the circumscribed, inscribed, and escribed circles of a $\triangle$, prove $r_1+r_2+r_3-r=4R$.

41. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find the length (x) of a rectangular parallelopiped $b=5$ ft. and $h=3$ ft., which can be *diagonally inscribed* in a similar parallelopiped $L=83$ ft., $B=64$ ft., and $H=50$ ft.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

26. Proposed by Professor J. F. W. SCHEFFER, M. A., Hagerstown, Maryland.

According to Bessel, the ratio of the squares of the polar diameter of the earth to that of the equatorial diameter, is .9933254. Find what *latitude* the angle made by a body falling to the earth, with a perpendicular to the surface, is greatest. Find, also, this maximum angle.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let ϕ=the required geographical latitude, and ϕ'=the geocentric latitude of the same place; then *Chauvenet's Spherical and Practical Astronomy*, Vol. I., p. 98, we deduce $\phi'=\tan^{-1}[(b^2/a^2)\tan\phi]=\tan^{-1}[(1-e^2)\tan\phi]$.

$$\therefore\quad (\phi-\phi')=\phi-\tan^{-1}[(1-e^2)\tan\phi], =\text{a Maximum}.$$

$$\therefore\quad \frac{d(\phi-\phi')}{d\phi}=1-\frac{(1-e^2)(1+\tan^2\phi)}{1+(1+e^2)^2\tan^2\phi}=0.$$

$$\therefore\quad \phi=\tan^{-1}\left[\sqrt{\left(\frac{1}{1-e^2}\right)}\right]=\tan^{-1}(1.0033541)=45^\circ\ 5'\ 45''.32,$$

$$\text{and } \phi'=\tan^{-1}(.9966571)=44^\circ\ 54'\ 14''.67.$$

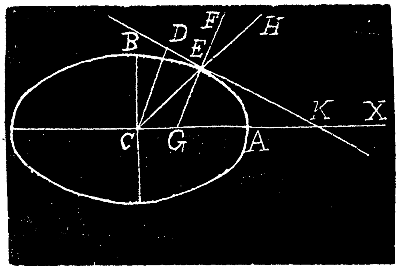
Hence $(\phi-\phi')=11'\ 30''.65$; and this result is found in the already-named Manual of Astronomy, Vol. II.; third Table, p. 577.

II. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Not taking into account the eastward deviation due to the rotation of the earth we can proceed as follows:

Let HEC be the direction the body falls, FEG the perpendicular to the earth's surface at E, DEK the tangent to the meridian at E, $CA=a$,

$CB=b$, $\angle ECA=\theta$, $\angle DCE=\angle CEG=\phi$, $\angle EKX=\beta$, co-ordinates of $E=(x,y)$.



Then $\tan\beta=-\dfrac{b^2x}{a^2y}$, $\tan\theta=\dfrac{y}{x}$, also

$\beta=90^\circ+\theta+\phi$, $\tan\beta=\tan(90^\circ+\theta+\phi)$

$$=-\cot(\theta+\phi).$$

$$\therefore\ \frac{b^2x}{a^2y}=\cot(\theta+\phi)=\frac{\cot\theta\cot\phi-1}{\cot\theta+\cot\phi}$$

$$=\frac{\dfrac{x}{y}\cot\phi-1}{\dfrac{x}{y}+\cot\phi}.$$

$$\therefore\ \tan\phi=\frac{a^2-b^2}{a^2b^2}xy=\text{maximum}\ldots.(1).\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ldots.(2).$$

The first differentials of (1) and (2) give $b^2x^2=a^2y^2$.

$\therefore\ x=\dfrac{a}{\sqrt{2}}$, $y=\dfrac{b}{\sqrt{2}}$, $\therefore\ \tan\theta=\dfrac{b}{a}=(.9933254)^{\frac{1}{2}}=.996659$, $\therefore\ \theta=44^\circ\ 54'$ $14''.9$=the latitude $\tan\phi=\dfrac{a^2-b^2}{2ab}=\dfrac{.0066746}{1.993318}=.003348$. $\therefore\ \phi=1'\ 30''.5$ =maximum angle made with the perpendicular.

Also solved by Professor *C. W. M. Black*, and the *Proposer*.

PROBLEMS.

36. Proposed by H. C. WHITAKER, B. Sc., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

A cube is revolved on its diagonal as an axis. Define the figure described and calculate its volume.

37. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

A man ties two mules—one to the outside of a circular wall, the other to the inside. If the lengths of the ropes of each is one-fourth the circumference of the wall, and both together can graze over one acre of ground: find the circumference of the wall.